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Entropy generation in combined heat and mass transfer

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Abstract—Irreversible entropy generation for combined forced convection heat and mass transfer in a two-dimensional channel is investigated. The heat and mass transfer rates are assumed to be constant on both channel walls. For the case of laminar flow, the entropy generation is obtained as a function of velocity, temperature, concentration gradients and the physical properties of the fluid. The analogy between heat and mass transfer is used to obtain the concentration profile for the diffusing species. The optimum plate spacing is determined, considering that either the mass flow rate or the channel length are fixed. For the turbulent flow regime, a control volume approach that uses heat and mass transfer correlations is developed to obtain the entropy generation and optimum plate spacing.

1. INTRODUCTION

IN RECENT years, thermal systems have been analyzed and optimized using the second law of thermodynamics. The present study deals with the irreversibilities that occur in combined forced convection heat and mass transfer in a two-dimensional channel flow. In order to minimize the irreversibilities in the system, the entropy generation is investigated [1, 2].

Bejan [3, 4] showed that the entropy generation for forced convective heat transfer in a channel is due to temperature gradient and viscosity effect in the fluid. For the case of heat transfer in a circular tube, Bejan demonstrated that when the entropy generation is minimized, a trade-off exists between temperature and viscosity effects. The concept of minimum entropy generation had also been applied to design a counterflow heat exchanger [5], and the optimum flow pathlength (the ratio of channel length to hydraulic diameter), was obtained.

Bejan [6] analyzed a sensible gas–liquid heat storage unit utilizing the second law. The optimum charging period and optimum number of transfer units were obtained for certain applications. The second-law performance of a two-dimensional fixed-bed regenerator, with finite wall heat conduction perpendicular to the mean flow direction was investigated by San [7]. He obtained the optimum Ntu and non-dimensional cycle time that yields the highest second-law efficiency for a regenerator. He also gave a general procedure to design a regenerator with fixed channel geometry. The entropy generation for the case of isothermal convective mass transfer was studied by San *et al.* [8], using the analogy between heat and mass transfer and considering Lewis number equal to unity and small mass diffusion rate.

In the present study, the general entropy generation equation is simplified to a form containing the temperature, velocity and mass concentration as variables for the case of fully-developed, two-dimensional channel flow. The mass diffusion rate is assumed to be small, and the heat transfer and mass diffusion rates are constant at the upper and lower walls. An order of magnitude analysis is performed for the energy equation in order to investigate the influence of mass diffusion on the temperature profile. For the case of laminar flow, the fully-developed velocity, temperature and mass concentration profiles are used to express the entropy generation in terms of the channel geometry and fluid properties. The entropy generation is then minimized to obtain the optimum channel geometry for a fixed heat and mass transfer loading. For the case of turbulent flow, the entropy generation and optimum channel geometry are determined using available correlations for heat transfer, mass transfer and friction data.

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NOMENCLATURE

$C, \Delta C$	concentration and concentration difference, $C_{A1} - C_c$ [mol m^{-3}]
c_p	constant pressure specific heat [J mol^{-1} K^{-1} or J kg^{-1} K^{-1}]
c_v	constant volume specific heat [J mol^{-1} K^{-1}]
$2D$	channel spacing [m or mm]
D_h	hydraulic diameter [m]
D_v	mass diffusivity [m^2 s^{-1}]
Ec	Eckert number, $U^2/c_{p,m}T_{AV}$
Eu_A	Euler number, $P_A/\rho_A U^2$
f	non-dimensional friction coefficient, $(dP/dx) = (f/D_h)(2G^2/\rho)$
g	rate of entropy generation per unit volume [J m^{-3} s^{-1} K^{-1}]
$\bar{g}$	rate of entropy generation per unit length and depth [J m^{-2} s^{-1} K^{-1}]
$\bar{\bar{g}}$	rate of entropy generation per unit depth [J m^{-1} s^{-1} K^{-1}]
G	mass flux [kg m^{-2} s^{-1}]
G_{eq}	equivalent mass flux, $\dot{q}''/c_{p,m}T_{AV}$ [kg m^{-2} s^{-1}]
h	convective heat transfer coefficient [W m^{-2} K^{-1}]
h_A	specific enthalpy of species A [J mol^{-1} or kJ kg^{-1}]
h_m	specific enthalpy of mixture [kJ (kg mixture)$^{-1}$]
H	enthalpy of mixture per unit depth [J m^{-1}]
J	mass diffusion flux [mol m^{-2} s^{-1}]
k	thermal conductivity [W m^{-1} K^{-1}]
k_v	mass transfer coefficient [m s^{-1}]
$\dot{\bar{K}}$	rate of formation per unit volume [mol m^{-3} s^{-1}]
L	length scale [m]
$\dot{m}$	mass flowrate per unit depth [kg m^{-1} s^{-1}]
$\dot{m}_d$	rate of mass diffusion [kg m^{-2} s^{-1}]
M	molecular weight [g mol^{-1}]
n_j	number of moles of specie j
Nu_D	Nusselt number, hD/k
P	local pressure [kPa]
Pe	Peclet number, $3U_{max}D/2$
Pr	Prandtl number, $\mu' c_p/k$
$\dot{q}''$	rate of heat transfer per unit depth and length [W m^{-2}]
R	gas constant [J mol^{-1} K^{-1}]
Re	Reynolds number, $\rho U D_h/\mu'$
s	specific entropy [J mol^{-1} K^{-1}]
S	entropy of mixture per unit depth [J m^{-1} K^{-1}]
$\bar{S}$	partial molar entropy of a substance [J mol^{-1} K^{-1}]
Sc_A	Schmidt number, $\mu'/\rho D_{v,A}$
Sh_D	Sherwood number, $k_v D/D_{v,A}$
St_h	heat transfer Stanton number, $h/Gc_{p,m}$
St_m	mass transfer Stanton number, $k_v \rho/G$
$T, \Delta T$	temperature and temperature difference, $T_1 - T_c$ [K]
U, U_{max}	fluid velocity and maximum fluid velocity [m s^{-1}]
V	volume [m^3]
x, y	coordinate notations
X	body force [J mol^{-1} m^{-1}]
X^*	normalized x-direction coordinate, x/L_x
Y^*	normalized y-direction coordinate, y/L_y or non-dimensional y-direction coordinate, y/D.

Greek symbols

α	thermal diffusivity or species [m^2 s^{-1}]
Δ	difference
ε	energy flux [J m^{-2} s^{-1}]
θ	normalized temperature, $(T - T_c)/(T_1 - T_c)$
μ'	dynamic friction coefficient of viscosity [kg m^{-1} s^{-1}]
μ	chemical potential [J mol^{-1}]
ρ	mean fluid density [kg m^{-3}]
τ	normalized concentration, $(C_A - C_c)/(C_{A1} - C_c)$.

Subscripts

0	reference state or wall surface
1	inlet condition
A	specie A
AV	average quantity
c	characteristic quantity
i, j	Cartesian tensor notations
m	air–water vapor mixture or mean value
x, y	coordinate notations.

Superscripts

·	rate.

2. LOCAL ENTROPY GENERATION IN COMBINED HEAT AND MASS TRANSFER

In a continuous flow field, the generation of entropy is due to the irreversible processes of heat transfer, mass transfer, momentum transfer (fluid friction), chemical reaction and the coupling between heat and mass transfer. The general formulation for the local entropy generation per unit volume, g, in an incompressible Newtonian fluid had been derived by

Hirschfelder *et al.* [9], and is given as

$$g = \frac{\mu'}{T}\left(\frac{\partial U_i}{\partial x_j}\right)\left[\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right] - \frac{1}{T}\left[\sum_\alpha J_{\alpha i}\left(\frac{\partial \mu_\alpha}{\partial x_i}\right)+\frac{\varepsilon_i}{T}\left(\frac{\partial T}{\partial x_i}\right) + \sum_\alpha \bar{S}_\alpha J_{\alpha i}\left(\frac{\partial T}{\partial x_i}\right) - \sum_\alpha J_{\alpha i}X_{\alpha i} + \sum_\alpha \bar{K}_\alpha \mu_\alpha\right] \quad (1)$$

where the first term is due to fluid friction and the second and third terms are due to mass diffusion and heat conduction. The fourth term arises from the coupling between heat and mass transfer, the fifth term is due to body forces, and the sixth term represents the effect due to chemical reactions.

Consider a two-dimensional channel flow with heat and mass transfer occurring at both walls. The chemical reactions and gravitational effects are neglected, and the fluid is considered to be a binary mixture of two ideal gases with species A diffusing perpendicular to the flow direction.

The chemical potential of species A can be expressed in the following form [10]:

$$\mu_A(T, P_A) = \mu_A^\circ(T) + RT\ln(P_A/P_0) \quad (2)$$

where P_A is the partial pressure of species A, and μ_A° is the standard state chemical potential of pure species A at temperature T. If $c_{p,A}$ is constant, $\mu_A^\circ(T)$ can be expressed as [11]

$$\mu_A^\circ(T) = c_{p,A}(T-T_0) - c_{p,A}T\ln\left(\frac{T}{T_0}\right) + h_{A0} - Ts_{A0} \quad (3)$$

where P_0 and T_0 are the reference pressure and temperature, and h_{A0} and s_{A0} are the enthalpy and entropy of the diffusing species, A, at T_0 and P_0. For example, let the reference state be the saturated vapor state of water at 373.1 K and 101.35 kPa. Using calorimetric data, the absolute entropy of liquid water at 3.169 kPa and 298.1 K is found to be equal to 66.68 J mol^{-1} K^{-1} [12]. The difference in entropy between liquid water at the state of 3.169 kPa, 298.1 K and saturated water vapor at the reference state is easily determined using the steam table to be 125.78 J mol^{-1} K^{-1}. Therefore, s_{A0} is equal to 192.46 J mol^{-1} K^{-1}.

Using equations (2) and (3), the partial molar entropy of species A, $\bar{S}_A$, can be simplified as

$$\bar{S}_A \equiv -\left(\frac{\partial \mu_A}{\partial T}\right)_{p,n_{j\neq A}} = -R\left(\ln\frac{C_A}{C_0}\right) + c_{v,A}\left(\ln\frac{T}{T_0}\right) + s_{A0} \quad (4)$$

where C_0 is the reference mass concentration at T_0 and P_0.

Substituting equations (2)–(4) into equation (1), the local entropy generation, g, in a two-dimensional channel flow with a single species, A, diffusing in the y-direction can be expressed as

$$g = \underbrace{\frac{\mu'}{T}\left\{2\left[\left(\frac{\partial U_x}{\partial x}\right)^2+\left(\frac{\partial U_y}{\partial y}\right)^2\right] + \left[\left(\frac{\partial U_x}{\partial y}\right)+\left(\frac{\partial U_y}{\partial x}\right)\right]^2\right\}}_{\text{(i)}} + \underbrace{R\left(\frac{D_{v,A}}{C_A}\right)\left(\frac{dC_A}{dy}\right)^2}_{\text{(ii)}} + \underbrace{\frac{k}{T^2}\left[\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right]}_{\text{(iii)}} + \underbrace{R\left(\frac{D_{v,A}}{T}\right)\left(\frac{dC_A}{dy}\right)\left(\frac{\partial T}{\partial y}\right)}_{\text{(iv)}} \quad (5)$$

where term (i) is due to fluid friction, term (ii) is due to mass diffusion, term (iii) is due to the flux of heat, and term (iv) is due to the coupling effect between heat and mass transfer.

3. ORDER OF MAGNITUDE ANALYSIS

In this section, the energy equation is normalized and the influence of mass diffusion on the temperature profile is determined for air–water vapor systems in a long two-dimensional channel.

3.1. *Energy equation*

Neglecting any change in potential energy in the flow field and assuming that the axial pressure gradient is small, the energy equation for the steady, two-dimensional flow of a mixture with a small diffusion rate of species A perpendicular to the direction of flow can be expressed as [13]

$$\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) - \rho U_x\left(\frac{\partial h_m}{\partial x}\right) - \rho U_y\left(\frac{\partial h_m}{\partial y}\right) + \frac{d}{dy}\left(h_A D_{v,A} M_A \frac{dC_A}{dy}\right) = 0 \quad (6)$$

where h_m is the specific enthalpy of the mixture and h_A is the specific enthalpy of the diffusing species, A.

Consider that the mixture is composed of ideal gases with constant k, $D_{v,A}$ and $c_{p,A}$. In the case of small mass diffusion rate, the constant pressure specific heat of the mixture, $c_{p,m}$, can also be assumed to be constant. Under these circumstances, equation (6) is simplified to

$$\alpha\frac{\partial^2 T}{\partial y^2} - U_x\frac{\partial T}{\partial x} - U_y\frac{\partial T}{\partial y} + \left(\frac{c_{p,A}}{c_{p,m}}\right)\left(\frac{M_A D_{v,A}}{\rho}\right)\left(\frac{\partial T}{\partial y}\right)\left(\frac{dC_A}{dy}\right) + \left(\frac{c_{p,A}}{c_{p,m}}\right)\left(\frac{M_A D_{v,A}}{\rho}\right)T\left(\frac{d^2 C_A}{dy^2}\right) = 0. \quad (7)$$

Equation (7) can be normalized into the following form:

$$\underset{\text{(i)}}{\left(\frac{\alpha \Delta T}{L_y^2}\right)\frac{\partial^2 \theta}{\partial Y^{*2}}} - \underset{\text{(ii)}}{\left(\frac{U_x \Delta T}{L_x}\right)\frac{\partial \theta}{\partial x^*}} - \underset{\text{(iii)}}{\left(\frac{U_y \Delta T}{L_y}\right)\frac{\partial 0}{\partial Y^*}}$$

$$+ \underset{\text{(iv)}}{\left(\frac{c_{p,\mathrm{A}}}{c_{p,\mathrm{m}}}\right)\left(\frac{M_{\mathrm{A}} D_{v,\mathrm{A}} \Delta T \Delta C}{\rho L_y^2}\right)\frac{\partial \theta}{\partial Y^*}\frac{\partial \tau}{\partial Y^*}}$$

$$+ \underset{\text{(v)}}{\left(\frac{c_{p,\mathrm{A}}}{c_{p,\mathrm{m}}}\right)\left(\frac{M_{\mathrm{A}} D_{v,\mathrm{A}} T \Delta C}{\rho L_y^2}\right)\frac{\partial^2 \tau}{\partial^2 Y^{*2}}} = 0 \quad (8)$$

where θ and τ are the normalized temperature and mass concentration, respectively. ΔT represents $(T_{\mathrm{I}} - T_{\mathrm{c}})$ and ΔC is $(C_{\mathrm{AI}} - C_{\mathrm{c}})$, which are the maximum temperature and concentration differences in the flow field. T_{I} and C_{AI} are the fluid temperature and the concentration of species A at the inlet, and T_{c} and C_{c} are the characteristic temperature and concentration, respectively. For example, in a two-dimensional channel flow, the characteristic temperature and concentration are the maximum (or minimum) values on the walls if heat and mass are transferred from the walls to the fluid (or vice versa).

In equation (8), both the temperature and concentration are normalized between 0 and 1. The normalized concentration profile is of the same order of magnitude as the normalized temperature profile for a mixture with Lewis number equal to unity. Therefore, each coefficient in equation (8) indicates the magnitude of each term. The first three terms in equation (8) govern the characteristics of the fully-developed temperature profile in a long channel, and they are assumed to have the same order of magnitude. The effect of mass diffusion is included in the next two terms in which the ratio $(c_{p,\mathrm{A}}/c_{p,\mathrm{m}})$ is close to 1.8 for water vapor diffusing into moist air. Comparing these two terms to the first term, we see that the two parameters, $[M_{\mathrm{A}} D_{v,\mathrm{A}} \Delta C/\alpha\rho]$ and

$$[(M_{\mathrm{A}} D_{v,\mathrm{A}} \Delta C/\alpha\rho)(T/\Delta T)],$$

determine the influence of mass diffusion on the temperature profile. As long as both terms are much smaller than unity, the temperature distribution and temperature gradients will not be significantly affected by the mass transfer. Under these circumstances, the fully-developed temperature profile for heat transfer alone can be used to calculate the entropy generation in combined heat and mass transfer.

In general, for the case of coupled heat and mass transfer in an air–water vapor system, the above two parameters are much less than unity. For instance, consider a long two-dimensional heat and mass exchanger with the following specifications: $T \sim 306$ K, $\rho = 1.165$ kg m^{-3}, $M_{\mathrm{A}} = 18$ g mol^{-1}, $D_{v,\mathrm{A}} = 26.1 \times 10^{-6}$ m^2 s^{-1}, $\Delta T \sim 15$ K, $\alpha = 22.5 \times 10^{-6}$ m^2 s^{-1} and $\Delta C \sim 0.0325$ mol m^{-3}. Using these data, we find

$$O(\text{term iv/term i}) \sim 5.8 \times 10^{-4} \qquad (9)$$

and

$$O(\text{term v/term i}) \sim 0.012. \qquad (10)$$

Therefore, the temperature profile of the pure heat transfer problem will be utilized in all calculations for entropy generation in combined heat and mass transfer presented here.

4. ENTROPY GENERATION IN COMBINED HEAT AND MASS TRANSFER—LAMINAR FLOW

In order to determine the entropy generation in a channel flow when heat and mass are being transferred simultaneously, the velocity, temperature and concentration fields must be known. If the flow is fully developed and laminar, the velocity, temperature and concentration fields are well known and can be directly used to evaluate the entropy generation. On the other hand, if the flow is developing, the resulting velocity, temperature and concentration fields must be determined either numerically or from available data. If the flow is turbulent only the root-mean-square value of the velocity, temperature and concentration fields are well known. Therefore, to determine the entropy generation in turbulent flow, a control volume approach is used. Bejan [14] and San [7] have shown that both approaches yield the same results for the case of laminar flow.

4.1. *Hydrodynamically fully-developed velocity profile*

Consider laminar flow of a gaseous mixture in a two-dimensional channel with a single species diffusing at a small rate perpendicular to the flow direction. Using these assumptions, with a plate spacing of $2D$, the fully-developed velocity profile is given as [15]

$$U_x(y) = 2U_{\max}\left[Y^* - \left(\frac{1}{2}\right)Y^{*2}\right] \qquad (11)$$

where U_y and $(\partial U_x/\partial x)$ are equal to zero.

4.2. *Fully-developed temperature profile*

As was discussed in Section 3.1, the order of magnitude analysis shows that as the non-dimensional groups $[M_{\mathrm{A}} D_{v,\mathrm{A}} \Delta C/\alpha\rho]$ and $[M_{\mathrm{A}} D_{v,\mathrm{A}} \Delta C/\alpha\rho][T_{\mathrm{AV}}/\Delta T]$ become much less than unity, the mass diffusion has little effect on the temperature profile. Therefore, for the condition of small mass diffusion rate, the solution of the convective heat transfer problem is used to determine the temperature profile in the combined heat and mass transfer problem.

The temperature distribution in the fluid considering the flow to be fully-developed with constant heat flux boundary conditions on both channel walls can be solved using the successive approximation method as described by Kays and Crawford [13]. The solution is as follows:

$$\frac{T-T_0}{T_m-T_0}=\frac{1}{0.486}\left[Y^*-\frac{1}{2}Y^{*3}+\frac{1}{8}Y^{*4}\right] \quad (12)$$

where T_m is the mean fluid temperature and T_0 is the surface wall temperature, both depend on the axial position within the channel.

4.3. *Analogy between heat and mass transfer*

The diffusion equation for species A has the same non-dimensional form as the energy equation for the mixture when mass diffusion effects are neglected. For diffusion of a substance with Lewis number equal to unity, the solution for the mass transfer problem is in the same form as the solution for the heat transfer problem, provided that the boundary conditions have the same form. Therefore, the concentration profile for the case of constant mass flux at both channel walls is given as

$$\frac{C_A-C_{A0}}{C_m-C_{A0}}=\frac{1}{0.486}\left[Y^*-\frac{1}{2}Y^{*3}+\frac{1}{8}Y^{*4}\right] \quad (13)$$

where C_m is the mean fluid mass concentration and C_{A0} is the surface mass concentration in the channel. These two variables depend on the position within the channel. In a heat and mass exchanger, where water vapor diffuses into air, the Lewis number is close to unity. Therefore, equation (13) is a valid expression for the mass concentration profile of water vapor in the channel.

4.4. *Entropy generation in fully-developed flow*

Substituting equations (11)–(13) into equation (5), the local entropy generation can be expressed as

$$\begin{aligned} g=&\left(\frac{4\mu' U_{\max}^2}{TD^2}\right)[1-Y^*]^2+\left(\frac{R\dot{m}_d^2}{D_{v,A}C_AM_A^2}\right)\\ &\times\left[1-\frac{3}{2}Y^{*2}+\frac{1}{2}Y^{*3}\right]^2+\left(\frac{\dot{q}''^2}{kT^2}\right)\left[\left(1-\frac{3}{2}Y^{*2}\right.\right.\\ &\left.\left.+\frac{1}{2}Y^{*3}\right)^2+\left(\frac{9.526}{Pe^2}\right)\left(Y^*-\frac{1}{2}Y^{*3}+\frac{1}{8}Y^{*4}\right)^2\right]\\ &+\left(\frac{R\dot{m}_d\dot{q}''}{kTM_A}\right)\left(1-\frac{3}{2}Y^{*2}+\frac{1}{2}Y^{*3}\right)^2. \end{aligned} \quad (14)$$

For small temperature and mass concentration variations in the channel, the entropy generation per unit depth and length, $\bar{g}$, can be obtained by integrating equation (14) across the channel width, $2D$. This yields

$$\begin{aligned} \bar{g}&\equiv\int_0^{2D} g\,\mathrm{d}y\\ &=\left(\frac{3}{2}\right)\left(\frac{\mu'\dot{m}^2}{T_{AV}\rho^2D^3}\right)+\left(\frac{34}{35}\right)\left(\frac{RD\dot{m}_d^2}{D_{v,A}C_{AV}M_A^2}\right)\\ &\quad+\left(\frac{34}{35}\right)\left(\frac{RD\dot{q}''\dot{m}_d}{kM_AT_{AV}}\right)+\left(\frac{34}{35}+\frac{3.75}{Pe^2}\right)\left(\frac{\dot{q}''^2D}{kT_{AV}^2}\right). \end{aligned} \quad (15)$$

In the above two equations, the Peclet number, Pe, gives the relative importance of the lateral transport (y-direction) to the axial transport (x-direction). Usually, the Peclet number for non-metallic substances is very large. For example, considering air at 300 K, $\alpha=22.5\times10^{-6}$ m^2 s^{-1}, $U_{\max}=1.15$ m s^{-1}, $D=0.00125$ m, the Peclet number is equal to 95.8. Therefore, in an air–water vapor system, those terms containing the Peclet number can be neglected. Using this result, equation (15) can be simplified as

$$\begin{aligned} \bar{g}=&\left(\frac{3}{2}\right)\left(\frac{\mu'\dot{m}^2}{T_{AV}\rho^2D^3}\right)+\left(\frac{34}{35}\right)\left(\frac{RD\dot{m}_d^2}{D_{v,A}C_{AV}M_A^2}\right)\\ &+\left(\frac{34}{35}\right)\left(\frac{\dot{q}''^2D}{kT_{AV}^2}\right)+\left(\frac{34}{35}\right)\left(\frac{RD\dot{q}''\dot{m}_d}{kM_AT_{AV}}\right). \end{aligned} \quad (16)$$

Equation (16) indicates that the entropy generation at any cross-section is independent of x for this case (constant heat flux). Using equation (16), the optimum design of a channel in which heat and mass transport occur simultaneously can be found by minimizing the total irreversible entropy generation.

4.5. *Optimum design criterion*

The four terms in equation (16) depend differently on the plate spacing, $2D$. The first term is inversely proportional to the cube of the channel spacing, while the others are linearly proportional to the channel spacing. As the plate spacing decreases, the velocity gradient will become larger, consequently, increasing the entropy generation due to fluid friction. Conversely, decreasing the plate spacing reduces the temperature and the mass concentration gradients, which in turn, reduces the entropy generation due to heat and mass transfer, provided that the heat and mass transfer rates still remain the same. Therefore, a plate spacing, $2D$, that minimizes the total entropy generation can be determined with all the other parameters fixed. This optimum plate spacing will give the optimum channel design for a required heat and mass transfer rate. Using equation (16), the optimum plate spacing, $2D$, can be expressed as

$$(2D)_{\text{opt}}=\left[\left(\frac{1260}{17}\right)\times\left(\frac{\dfrac{\mu'\dot{m}^2}{T_{AV}\rho^2}}{\dfrac{\dot{m}_d^2R}{D_{v,A}C_{AV}M_A^2}+\dfrac{\dot{q}''^2}{kT_{AV}^2}+\dfrac{\dot{q}''\dot{m}_dR}{kT_{AV}M_A}}\right)\right]^{1/4}. \quad (17)$$

4.6. *Two-dimensional entropy generation profile*

Using equation (14) and neglecting the axial conduction terms, we see that the local entropy generation is minimum at the center of the channel, i.e.

$$\text{at } Y^*=1.0, \quad g_{\min}=0. \quad (18)$$

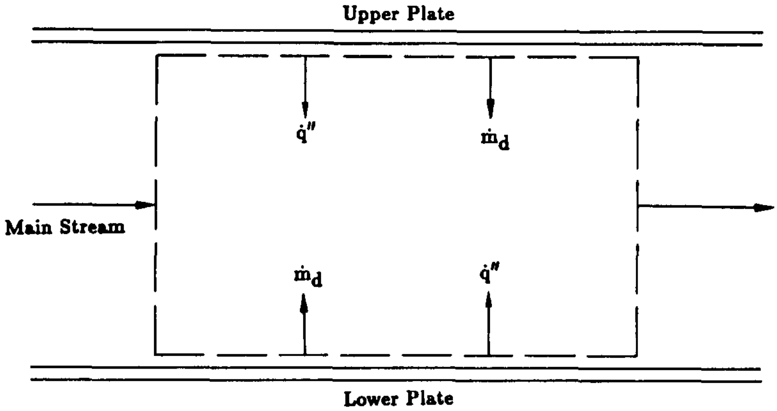

FIG. 1. Two-dimensional model.

This result is due to the symmetry of the velocity, temperature and mass concentration profiles at the center of the channel, and it results in no heat transfer and mass diffusion across the centerline. Similarly, the location of the maximum entropy generation occurs at the surfaces of the channel. The numerical value can be determined using the following expression:

at $Y^* = 0$ and 2.0

$$g_{\max} = \left(\frac{9}{4}\right)\left(\frac{\mu'\dot{m}^2}{\rho^2 D^4 T_{AV}}\right) + \frac{R\dot{m}_d^2}{D_{v,A}C_A M_A^2} + \frac{\dot{q}''^2}{kT_{AV}^2} + \frac{R\dot{m}_d\dot{q}''}{kT_{AV}M_A}. \quad (19)$$

4.7. *Numerical example—heat and mass exchanger*

Consider a two-dimensional channel in which heat and mass are transferred simultaneously (Fig. 1). Let both heat and water vapor diffuse at a constant rate from channel surfaces and let the total mass flowrate of air per unit depth passing through the exchanger be 0.0066 kg m^{-1} s^{-1}. The constant heat flux and mass diffusion rate are given as follows: $\dot{q}'' = -70.86$ W m^{-2}, $\dot{m}_d = 2.29\times10^{-5}$ kg m^{-2} s^{-1}. The minus sign indicates that the heat flux is transferred from the control surface to the surroundings. For air: $\rho = 1.165$ kg m^{-3}, $\mu' = 1.8\times10^{-5}$ kg m^{-1} s^{-1}, $k = 26.37\times10^{-3}$ W m^{-1} K^{-1}. For water vapor: $M_A = 18$ g mol^{-1}, $D_{v,A} = 26.1\times10^{-5}$ m^2 s^{-1}. The average fluid temperature, T_{AV}, is 306 K. The average concentration of water vapor, C_{AV}, calculated using the ideal gas equation of state is 1.03 mol m^{-3}.

The Reynolds number based on the hydraulic radius for this problem is 733, verifying that the flow is in the laminar flow regime. Using equation (17), the optimum plate spacing, $2D$, can be found as follows (Fig. 2):

$$(2D)_{\text{opt}} = 2.75\ \text{mm}. \quad (20)$$

The total entropy generation per unit depth and length in the channel is equal to 4.35×10^{-3} J m^{-2} s^{-1} K^{-1}. The fraction of entropy generation due to friction is 25.1%, due to the mass diffusion is 15.3%, and due to the heat transfer is 62.5%. The coupling effect due to simultaneous heat and mass transport reduces the total entropy generation by 2.9% (Fig. 3). In the case of convective heat transfer or isothermal convective mass transfer, respectively, the concentration or temperature is uniform in the flow field. Thus, the coupling effect in either case will be equal to zero.

The non-dimensional temperature and mass concentration variations respectively are evaluated as

$$\left(\frac{\Delta T}{T_{AV}}\right) < 5\%, \quad \frac{\Delta C}{C_{AV}} < 3.5\%. \quad (21)$$

Therefore, the assumption of small temperature and mass concentration variations is reasonable. Also, it is found that the two parameters governing the mass diffusion effect in the energy equation have the following order of magnitude

$$\left[\left(\frac{M_A D_{v,A}\Delta C}{\alpha\rho}\right)\left(\frac{T}{\Delta T}\right)\right] < 1.5\%,$$

$$\left[\frac{M_A D_{v,A}\Delta C}{\alpha\rho}\right] < 0.1\%. \quad (22)$$

Hence, the analogy between heat and mass transfer is suitable for this problem.

4.8. *Turbulent and/or developing flow*

4.8.1. *Control volume approach.* There is no exact solution for the velocity, temperature and mass concentration distributions for turbulent channel flow. Therefore, a control volume analysis, based on the average fluid properties over the channel cross-sectional area, is developed and solved using the results of turbulent flow correlations. The control volume is shown in Fig. 4. The main stream is a binary mixture with a single species, A, diffusing from the planar walls into the main stream. The simultaneous exchange of heat and mass between the boundaries and the main

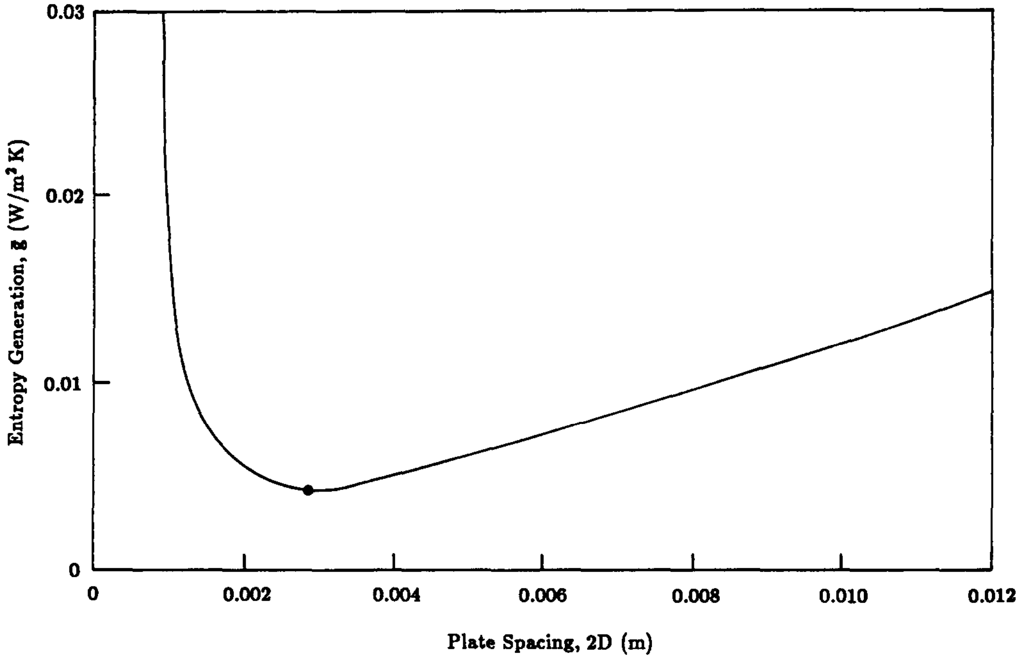

FIG. 2. Entropy generation vs plate spacing.

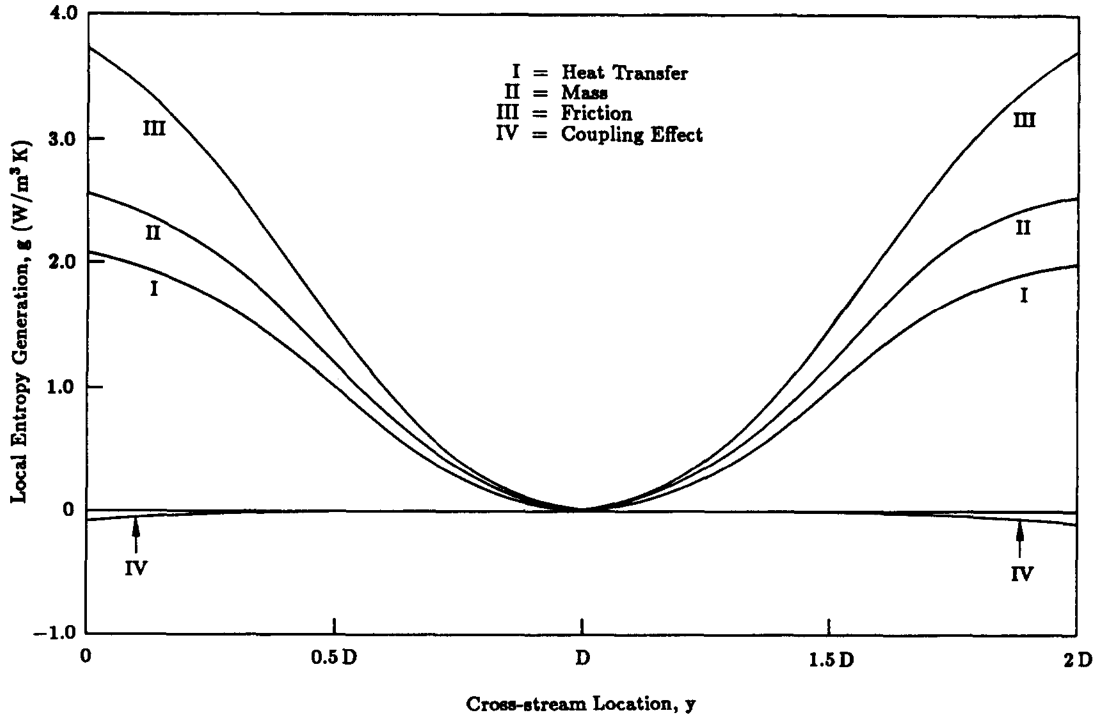

FIG. 3. Distribution of local entropy generation in the channel.

stream causes a continuous non-equilibrium field of temperature, pressure and chemical potential in the flow, and thus an irreversible generation of entropy. In order to simplify the analysis, the present study is based on the two-dimensional model as considered previously.

Applying the second law of thermodynamics to the control volume, we obtain

$$\bar{g} = \mathrm{d}\dot{S} - 2(\dot{m}_\mathrm{d}/M_\mathrm{A})s_\mathrm{A}\,\mathrm{d}x - \frac{2\dot{q}''\,\mathrm{d}x}{T_\mathrm{AV}+\Delta T} \qquad (23)$$

where $\mathrm{d}\dot{S}$ is the total rate of increase in entropy as the

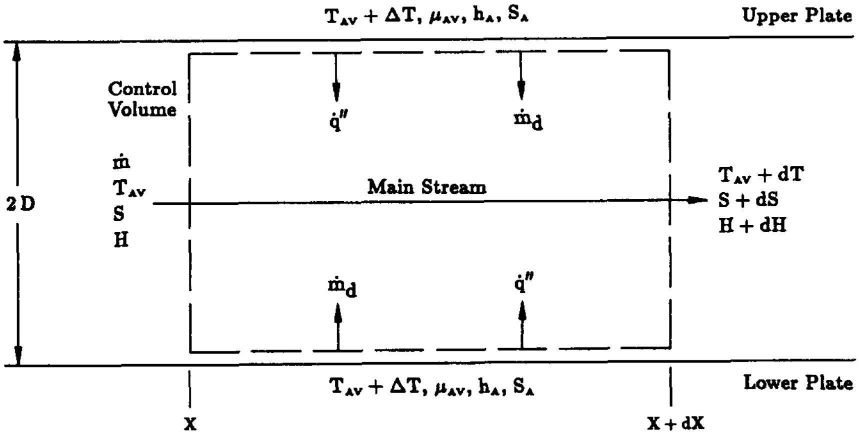

FIG. 4. Control volume approach in combined heat and mass transfer.

fluid flows through the small segment dx, the second term on the right-hand side of the equation is the total entropy of the substance diffusing into the control volume and the last term is the entropy increase due to the heat transfer occurring across the control surface, which is at temperature $(T_{AV}+\Delta T)$. T_{AV} is the mean fluid temperature and ΔT is the temperature difference between the mean fluid and the control surface.

Applying the first law of thermodynamics to the control volume, we obtain

$$d\dot{H} = 2\dot{q}''\,dx + 2(\dot{m}_d/M_A)h_A\,dx \qquad (24)$$

where $d\dot{H}$ is the total rate of increase in enthalpy of the mixture and h_A is the specific enthalpy of species A as it enters the control volume.

The Gibbs' equation for a binary fluid mixture at a temperature, T_{AV}, can be written as

$$dH = T_{AV}\,dS + V\,dp + \sum_{i \le 1}^{2} \mu_{AV,i}\,dn_i. \qquad (25)$$

Combining equations (23)–(25), the entropy generation can be expressed as

$$\bar{g} = \left(\frac{2\dot{q}''\,dx}{T_{AV}}\right)\left[1 - \frac{1}{1+\dfrac{\Delta T}{T_{AV}}}\right] - \left(\frac{\dot{V}}{T}\right)dP + \left(\frac{2\dot{m}_d\,dx}{M_A}\right)\left(\frac{h_A}{T_{AV}} - s_A - \frac{\mu_{AV}}{T_{AV}}\right). \qquad (26)$$

Using the ideal gas equation of state and the definition of chemical potential yields

$$\left[\frac{h_A}{T_{AV}} - s_A - \frac{\mu_{AV}}{T_{AV}}\right] = c_{p,A}\left(\frac{\Delta T}{T_{AV}}\right) - c_{v,A}\ln\left(1+\frac{\Delta T}{T_{AV}}\right) + R\ln\left(\frac{C_A}{C_{AV}}\right). \qquad (27)$$

For the case of small temperature and mass concentration variations in the flow field, equation (27) can be further simplified using the Taylor series expansion. Neglecting the higher order terms of $\Delta T/T_{AV}$ and $\Delta C/C_{AV}$, it yields

$$\left[\frac{h_A}{T_{AV}} - s_A - \frac{\mu_{AV}}{T_{AV}}\right] \approx R\left(\frac{\Delta T}{T_{AV}}\right) + R\left(\frac{\Delta C}{C_{AV}}\right). \qquad (28)$$

Substituting equation (28) into equation (26) and dividing $\bar{g}$ by dx. The entropy generation (per unit length and depth), $\bar{g}$, can be expressed as

$$\bar{g} = \left(\frac{2\dot{q}''}{T_{AV}}\right)\left(\frac{\Delta T}{T_{AV}}\right) + \left(\frac{2\dot{m}_d R}{M_A}\right)\left(\frac{\Delta C}{C_{AV}}\right) + \left(\frac{2\dot{m}_d R}{M_A}\right)\left(\frac{\Delta T}{T_{AV}}\right) - \left(\frac{\dot{V}}{T}\right)\left(\frac{dp}{dx}\right). \qquad (29)$$

Using the definitions of Nusselt and Sherwood numbers, ΔT and ΔC can be eliminated from equation (29). Also, consistent with the assumption of small diffusion rate, the pressure gradient term can be approximately expressed as: $(dP/dx) \sim (fm^2/8\rho L^3)$. Therefore, the entropy generation (per unit length and depth) takes on the following form:

$$\bar{g} = \underset{\text{(i)}}{\frac{2}{Nu_D}\left[\frac{\dot{q}''^2 D}{kT_{AV}^2}\right]} + \underset{\text{(ii)}}{\frac{2}{Sh_D}\left[\frac{\dot{m}_d^2 RD}{D_{v,A}C_{AV}M_A^2}\right]} + \underset{\text{(iii)}}{\frac{2}{Nu_D}\left[\frac{\dot{m}_d\dot{q}''RD}{kM_A T_{AV}}\right]} + \underset{\text{(iv)}}{\left(\frac{3}{2}\right)\left[\frac{\mu'\dot{m}^2}{\rho^2 D^3 T_{AV}}\right]} \qquad (30)$$

where in this result term (i) represents the heat transfer effect, term (ii) is the mass diffusion effect, term (iii) is due to the coupling effect between heat and mass transfer and term (iv) is the fluid friction effect. No other assumptions have been made in equation (30) except small mass diffusion rate and small temperature and mass concentration variations in the flow field. Hence, equation (30) can be applied to laminar and turbulent flows as well as to a developing flow at the entrance of a channel.

The relative magnitude of the four terms in equation (30) is obtained by comparing the mass diffusion, the coupling and the fluid friction effects to the heat transfer effect. We obtain the following three expressions:

$$\bar{g}_{\mathrm{mass}}/\bar{g}_{\mathrm{heat}} = (St_h/St_m)Eu_{\mathrm{A}}\,Ec(\rho/\rho_{\mathrm{A}})(\dot{m}_{\mathrm{d}}/G_{\mathrm{eq}})^2 \quad (31)$$

$$\bar{g}_{\mathrm{coupling}}/\bar{g}_{\mathrm{heat}} = (1-c_{v,\mathrm{m}}/c_{p,\mathrm{m}})(\dot{m}_{\mathrm{d}}/G_{\mathrm{eq}}) \quad (32)$$

$$\bar{g}_{\mathrm{friction}}/\bar{g}_{\mathrm{heat}} = (3/4)(St_h\,Ec/Re)(G/G_{\mathrm{eq}}) \quad (33)$$

where G is the mass flux of the mixture and G_{eq} is the equivalent mass flux which is defined as: $(\dot{q}''/c_{p,\mathrm{m}}T_{\mathrm{AV}})$. The equivalent mass flux is a fictitious quantity indicating the strength of the heat flux on the boundary surfaces. In equations (31) and (32), the mass diffusion and coupling effects are compared to the heat transfer effect respectively, thus the ratio of the mass diffusion rate to the equivalent mass flux is used to indicate the relative magnitude between the heat flux and mass diffusion rate. In equation (33), the fluid friction effect is compared to the heat transfer effect, thus the ratio of the mass flux in the main stream to the equivalent mass flux is used to indicate the relative magnitude. For air–water vapor systems, the heat transfer Stanton number is close to the mass transfer Stanton number, $(St_h/St_m) \sim 1.0$. The density ratio, (ρ/ρ_{A}), is of the order of 100. The product, $Eu_{\mathrm{A}}\,Ec$, is close to 0.46 and the ratio, $c_{v,\mathrm{m}}/c_{p,\mathrm{m}}$, is 0.72. Hence, the coupling effect is generally small as compared to the other effects.

As the non-dimensional groups $[M_{\mathrm{A}}D_{v,\mathrm{A}}\,\Delta C/\alpha\rho]$ and $[M_{\mathrm{A}}D_{v,\mathrm{A}}\,\Delta C/\alpha\rho][T_{\mathrm{AV}}/\Delta T]$ become much less than unity, the fully-developed temperature profile tends to be independent of mass transfer. Further, by neglecting the viscous dissipation in the energy equation, the analogy between heat and mass transfer is valid. Under these circumstances, the corresponding Nusselt and Sherwood numbers are constant and the optimum plate spacing, $2D$, can be expressed as

$$(2D)_{\mathrm{opt}} = \left\{(36)\left(\frac{\mu'\dot{m}^2}{\rho^2 T_{\mathrm{AV}}}\right)\middle/\left[\frac{1}{Nu_D}\left(\frac{\dot{q}''^2}{kT_{\mathrm{AV}}^2}\right)\right.\right.$$
$$\left.\left.+\frac{1}{Sh_D}\left(\frac{\dot{m}_{\mathrm{d}}^2 R}{D_{v,\mathrm{A}}C_{\mathrm{AV}}M_{\mathrm{A}}^2}\right)+\frac{1}{Nu_D}\left(\frac{\dot{m}_{\mathrm{d}}\dot{q}''R}{kM_{\mathrm{A}}T_{\mathrm{AV}}}\right)\right]\right\}. \quad (34)$$

For the case of constant heat flux and constant rate of mass diffusion in a laminar flow with Lewis number equal to unity, both the Nusselt and Sherwood numbers in equation (34) are equal to 2.058 and yield the same optimum plate spacing as equation (17).

For turbulent flow problems, we may substitute the Nusselt and Sherwood numbers based on the half channel spacing, D, into equation (34) with the following turbulent correlations [13]:

$$Nu_D = 0.0055Pr^{0.5}\,Re^{0.8} \quad 0.5 < Pr < 1.0 \quad (35)$$

$$Sh_D = 0.0055Sc_{\mathrm{A}}^{0.5}\,Re^{0.8} \quad 0.5 < Sc_{\mathrm{A}} < 1.0. \quad (36)$$

Equations (35) and (36) are valid for gas in the range of $3\times10^4 < Re < 10^5$.

4.8.2. *Finite temperature and mass concentration variation case.* As the temperature variation and the mass concentration variation increase, equation (30) will yield an error due to the linearization in equation (28). If both $(\Delta T/T_{\mathrm{AV}})$ and $(\Delta C/C_{\mathrm{AV}})$ are less than unity and the mass diffusion rate is still small, a correction of equation (34) which yields the optimum plate spacing is developed by including more terms in the Taylor series expansion. For example, in general equation (27) can be expressed as

$$\left[\frac{h_{\mathrm{A}}}{T_{\mathrm{AV}}}-S_{\mathrm{A}}-\frac{\mu_{\mathrm{AV}}}{T_{\mathrm{AV}}}\right]=c_{p,\mathrm{A}}\left(\frac{\Delta T}{T_{\mathrm{AV}}}\right)$$
$$-c_{v,\mathrm{A}}\sum_{n=1}^{n}\left[\frac{(-1)^{n+1}}{n!}\left(\frac{\Delta T}{T_{\mathrm{AV}}}\right)^n\right]$$
$$+R\sum_{n=1}^{n}\left[\frac{(-1)^{n+1}}{n!}\left(\frac{\Delta C}{C_{\mathrm{AV}}}\right)^n\right]. \quad (37)$$

Using the relation

$$\left[1-\frac{1}{1+\left(\dfrac{\Delta T}{T_{\mathrm{AV}}}\right)}\right]=\sum_{n=0}^{n}\left[\frac{(-1)^n}{n!}\left(\frac{\Delta T}{T_{\mathrm{AV}}}\right)^{n+1}\right]. \quad (38)$$

Substituting equations (37) and (38) into equation (26), and again introducing the Nusselt and Sherwood numbers, the entropy generation can be expressed as

$$\bar{g}=\underbrace{\sum_{n=0}^{n}\left\{\frac{(-1)^n(2)}{n!}\left(\frac{\dot{q}''}{T_{\mathrm{AV}}}\right)^{n+2}\left[\frac{D}{k\,Nu_D}\right]^{n+1}\right\}}_{\text{(i)}}$$
$$+\underbrace{\left(\frac{2c_{p,\mathrm{A}}\dot{m}_{\mathrm{d}}}{M_{\mathrm{A}}}\right)\left(\frac{\dot{q}''}{T_{\mathrm{AV}}}\right)\left[\frac{D}{k\,Nu_D}\right]}_{\text{(ii)}}$$
$$-\underbrace{\sum_{n=1}^{n}\left\{\frac{(-1)^{n+1}}{n!}\left(\frac{2c_{v,\mathrm{A}}\dot{m}_{\mathrm{d}}}{M_{\mathrm{A}}}\right)\left(\frac{\dot{q}''}{T_{\mathrm{AV}}}\right)^n\right.}_{\text{(iii)}}$$
$$\left.\times\left[\frac{D}{k\,Nu_D}\right]^n\right\}+\underbrace{\sum_{n=1}^{n}\left\{\frac{(2R)(-1)^{n+1}}{n!}\left(\frac{\dot{m}_{\mathrm{d}}}{M_{\mathrm{A}}}\right)^{n+1}\right.}_{\text{(iv)}}$$
$$\left.\times\left[\frac{D}{D_{v,\mathrm{A}}C_{\mathrm{AV}}Sh_D}\right]^n\right\}+\underbrace{\left(\frac{3}{2}\right)\left(\frac{\mu'\dot{m}^2}{T_{\mathrm{AV}}\rho^2D^3}\right)}_{\text{(v)}} \quad (39)$$

where now in equation (39), term (i) is the heat transfer effect, terms (ii) and (iii) are due to the coupling effect, term (iv) represents the mass diffusion effect and term (v) is the fluid friction effect. Choosing an appropriate number of terms in the series expansions, the optimum plate spacing can be solved numerically.

5. CONCLUSIONS

The entropy generation due to mass diffusion has a similar form to that due to heat transfer, and both are linearly dependent on the channel spacing. The relative magnitude between the mass diffusion effect and the heat transfer effect depends on $(St_h/St_m)Eu_A \times Ec(\rho/\rho_A)(\dot{m}_d/G_{eq})^2$ in which the equivalent mass flux indicates the strength of the heat flux. The coupling effect is small as compared to the other effects and it can be a negative value when the heat transfer is opposite in direction to the mass transfer.

As the heat flux or mass diffusion rate approaches zero, the entropy generation and optimum plate spacing for the process of combined heat and mass transfer reduce to the results for the processes of convective heat transfer and isothermal convective mass transfer, respectively. In either case, the coupling effect will approach zero. At plate spacing less than the optimum value, the entropy generation increases sharply. However, at plate spacing greater than the optimum value, the entropy generation increases only gradually.

The optimum plate spacing derived in this work is based on small temperature and concentration variations in the flow field. However, in the case of finite temperature and concentration variations, the presented series expansion method can be used to obtain the optimum plate spacing.

In the case of laminar flow, since both the Nusselt and Sherwood numbers are constant, the four different sources of entropy generation are coupled only through the plate spacing. On the other hand, in the case of turbulent flow, the four sources of entropy generation are coupled through both the plate spacing and Reynolds number.

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CREATION D'ENTROPIE DANS LE TRANSFERT COMBINE DE CHALEUR ET DE MASSE

Résumé—On étudie la création irréversible d'entropie pour la convection combinée de chaleur et de masse dans un canal bidimensionnel dont les deux parois sont à flux uniforme de chaleur et de masse. Dans le cas de l'écoulement laminaire, la création d'entropie est obtenue en fonction des gradients de vitesse, de température, et de concentration et des propriétés du fluide. L'analogie entre transfert de chaleur et de masse est utilisée pour obtenir le profil de concentration des espèces qui diffusent. On détermine l'espacement optimum des parois en considérant fixé le débit ou la longueur du canal. Pour le régime turbulent, on développe l'approche par volume de contrôle qui utilise les formules de transfert de chaleur et de masse et on obtient la création d'entropie et l'espacement optimal.

ENTROPIEERZEUGUNG BEIM GEKOPPELTEN WÄRME- UND STOFFTRANSPORT

Zusammenfassung—Es wurde die Entropieerzeugung beim gekoppelten Wärme- und Stofftransport in erzwungener Konvektion in einem zweidimensionalen Kanal untersucht. Die Wärme- und Stoffübertragung wird an beiden Kanalwänden als konstant angenommen. Für den Fall laminaren Strömung erhält man die Entropieerzeugung als eine Funktion von Geschwindigkeit, Temperatur, Konzentrationsgradienten und den physikalischen Eigenschaften des Fluids. Die Analogie zwischen Wärme- und Stoffübertragung wird dazu benutzt, die Konzentrationsprofile zu ermitteln. Der optimale Plattenabstand wird unter der Bedingung, daß entweder der Massendurchsatz oder die Kanallänge konstant gehalten wird, bestimmt. Für turbulente Strömung wird eine Näherung mit Hilfe eines Kontrollvolumens, das Wärme- und Stoffübertragungsbeziehungen benutzt, entwickelt, um die Entropieerzeugung und den optimalen Plattenabstand zu ermitteln.

ПРОИЗВОДСТВО ЭНТРОПИИ ПРИ СОВМЕСТНОМ ТЕПЛО-И МАССОПЕРЕНОСЕ

Аннотация—Исследуется производство неравновесной энтропии для случая вынужденноконвективного тепло-и массопереноса в двумерном канале, когда потоки тепла и массы постоянны на обеих стенках канала. При ламинарном течении найдено производство энтропии как функции градиентов скорости, температуры и концентрации, а также физических свойств жидкости. Для получения профиля концентрации диффундирующих веществ используется аналогия между теплопереносом и массопереносом. При заданных массовом расходе либо длине канала определено оптимальное расстояние между пластинами. Для турбулентного режима разработан способ измерения объема, в котором используется связь между переносом тепла и переносом массы для определения производства энтропии и оптимального расстояния между пластинами.